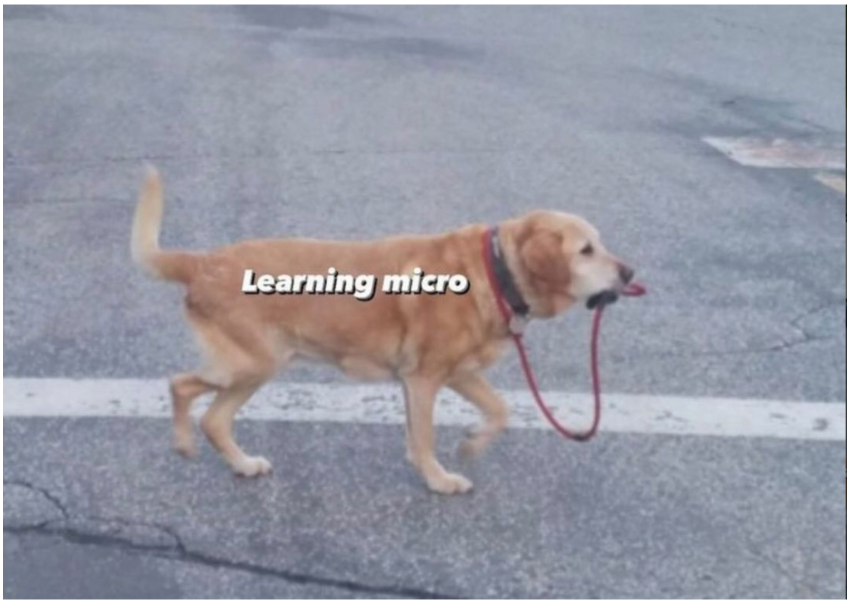
SOLVED EXAMS 2014-2020

Game theory guides



Solved exams 2014-2020
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Table of Contents

2020 exam – solved

The market for smoked salmon in Denmark is perfectly competitive. The inverse demand is $p = 10 - Q$. The inverse supply is $p = Q$. In this market the government imposes on consumers a per unit tax of 2.

A Find the optimal quantity produced in Denmark with the tax. Find the price consumers pay, and the price that producers pocket.

To find the optimal price and quantity. Find the initial demand curves

$$P + 2 = 10 - Q_d$$
$$Q_d = 8 - P$$

$$P = Q$$
$$Q = P$$

Set $Q_s = Q_d$

$$8 - P = P$$
$$P = 4$$

The price producers' pocket is $\boldsymbol{P_p^* = 4}$

$$Q_s(P) = 4$$

The optimal production is $\boldsymbol{Q^* = 4}$

$$P_d(Q) = 10 - 4 = 6$$

The price consumers pay is 6

B Find what percentage of the tax is paid by consumers in Denmark (hint: what price consumers pay without taxes?). Find the tax revenues for the Danish government.

Tax incidence by consumers

$$P_{no\ tax} = Q_s$$
$$Q_s = Q_d$$
$$10 - Q = Q$$
$$Q = 5$$

$$\frac{\Delta P}{\Delta T} = \frac{(P_{tax} - P_{no\,tax})}{tax} = \frac{6-5}{2} = 0{,}5 = 50\%$$

To find tax revenue

$$TR = tax * Q^* = 2 * 4 = 8$$

The tax revenue for the danish government is 8

Now, the market for smoked salmon in Jamaica is also perfectly competitive. The inverse demand in Jamaica is $p = 10 - 3Q$. The inverse supply is the same as in Denmark, $P = Q$. Also in Jamaica the government imposes on consumers a per unit tax of 2.

C Find the optimal quantity produced in Jamaica with the tax. Find the price consumers pay, and the price that producers pocket.

To find the optimal price and quantity

Find the initial demand curves

$$P + 2 = 10 - 3Q_d$$
$$3Q_d = 8 - P$$
$$Q_d = 2{,}67 - \frac{1}{3}P$$

$$P = Q$$
$$Q = P$$

Set $Q_s = Q_d$

$$2{,}67 - \frac{1}{3}P = P$$
$$\frac{4}{3}P = 2{,}67$$
$$P = 2$$

The price producers' pocket is $\boldsymbol{P_p^* = 2}$

Optimal production

$$Q_s(P = 2) = 2$$

The optimal production is $\boldsymbol{Q^* = 2}$

Producers pay:

$$P_d(Q = 2) = 10 - 3(2) = 4$$

The price consumers pay is $\boldsymbol{P_c^* = 4}$

D Find what percentage of the tax is paid by consumers in Jamaica (hint: what price consumers pay without taxes?). Find the tax revenues for the Jamaican government.

Tax incidence by consumers

$$P_{no\ tax} = Q_s = Q_d$$
$$10 - 3Q = Q$$
$$Q = 2{,}5$$

$$\frac{\Delta P}{\Delta T} = \frac{(P_{tax} - P_{no\,tax})}{tax} = \frac{4 - 2{,}5}{2} = 0{,}75 = 75\%$$

To find tax revenue

$$TR = tax * Q^* = 2 * 2 = 4$$

The tax revenue for the Jamaican government is 4

E Which consumers face a higher tax incidence, Jamaican consumers of Danish consumers? Why?

Jamaican consumers face a higher tax incidence as they pay 75% compared to 50% for danish consumers. This is because the Jamaican demand curve is steeper than the danish when looking at the coefficient (3 > 1)

Question 2

Emma has utility over money characterized by the following function: $U(Y) = Y^2$.

A Calculate the first and second order derivatives of the Utility function. Based on these calculations, is Emma risk averse, risk neutral, or risk loving?

$$U(Y)' = 2Y$$
$$U(Y)'' = 2$$

Emma is risk loving, as both the first- and second order derivative is positive

B Emma finds a lottery ticket on the street. The lottery ticket pays 10dkk with 80% probability, and 2000dkk with 20% probability. (a) Calculate the expected value (EV) of this lottery ticket. (b) Calculate the expected utility of Emma from this lottery (i.e. EU). (c) Calculate the utility for Emma from having an amount equal to the expected value (i.e. U(EV)).

a) The expected value

$$EV = 0{,}20(2000) + 0{,}8(10) = 408DKK$$

b) The expected utility

$$EU = 0{,}20(2000)^2 + 0{,}8(10)^2 = 800.080$$

c) Find the utility of the expected value

$$U(EV) = (408)^2 = 166.464$$

C Daniel, a friend of Emma, is thinking of offering Emma some money in exchange for the lottery ticket. (a) What amount of money should Daniel offer Emma for her to consider the offer? (b) What risk profile do you think Daniel should have in order for him to be willing to make that exchange with Emma?

a) What Daniel should pay

$$CE = EU^{-1} = \sqrt{EU} = \sqrt{800.080} = 894{,}47$$

Daniel must offer her at least 994,47 DKK for her to consider the offer

b) What Risk profile does Daniel have?

Daniels Willingness to Pay must be equal or higher than Emma's certainty equivalent for him to buy the lottery ticket. His risk profile must therefore be similar to Emma's, and he is therefore risk loving.

Emma decides to donate her ticket to her daughter Lily. Lily has utility over money characterized by the following function: $U_L(Y) = 4 \cdot Y$.

D Is Lily risk neutral, risk averse, or risk loving? Calculate the expected utility of Lily from this lottery (i.e. EU). Calculate the utility for Lily from having an amount equal to the expected value (i.e. U(EV)).

a) Risk profile

$$U_L(Y) = 4 * Y$$
$$U_L(Y)' = 4$$
$$U_L(Y)'' = 0$$

Lilly is risk neutral as it is linear function

b) The expected value

$$EV = 0{,}20(2000) + 0{,}8(10) = 408DKK$$

c) The expected utility

$$EU = 4 * (0{,}20(2000) + 4 * (0{,}8(10) = 1632$$

d) Find the utility of the expected value

$$U(EV) = 4(408) = 1632$$

E What amount of money should Daniel offer this time to Lily for her to consider the offer?

$$CE = EU^{-1} = \frac{EU}{4} = \frac{1632}{4} = 408DKK$$

As Lily is risk neutral, he should just offer her the expected value of the lottery ticket, 408 DKK

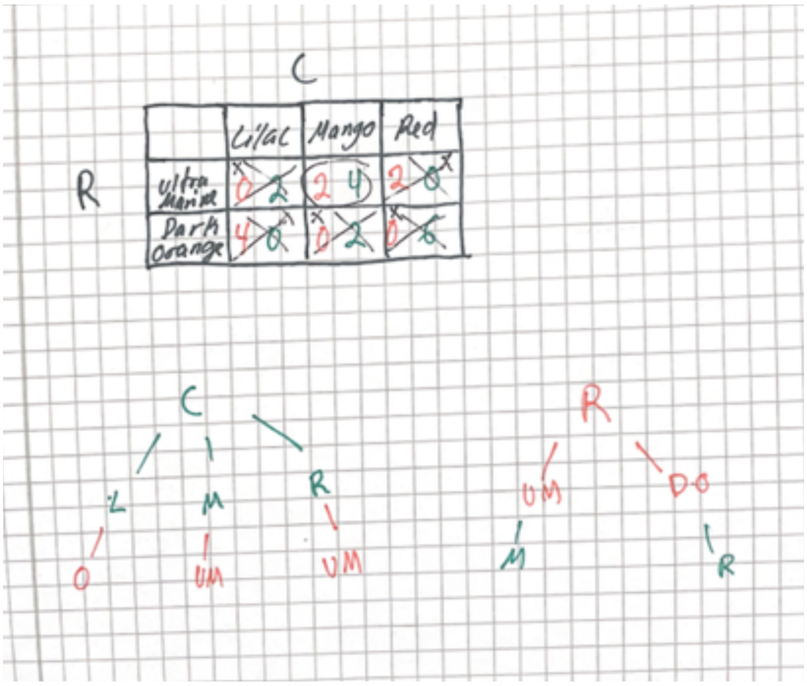
Question 3

Rebecca and Camilla are going to a party and must choose how to dress without knowing how the other will be dressed. Rebecca has two dresses to choose from (Ultramarine, Dark orange), and Camilla has three dresses to choose from (Lilac, Mango, Red). The simultaneous game can be described by the following matrix.

		Camilla		
		Lilac	Mango	Red
Rebecca	Ultramarine	0,2	2,4	2,0
	Dark Orange	4,0	0,2	0,6

A How many Nash Equilibria in pure strategies exist in this game? Find it/them.

There's one Nash Equilibrium ($Ultra\ Marine; Mango$)



B When some actions are dominated, we can solve the same problem by iterative elimination of dominated strategies (IEDS). Do Rebecca or Camilla have any actions that are initially dominated? If so, find the N.E. by using IEDS. Please show all the logical steps. (hint: step 1: one player has action X dominated. Step 2: if the other player knows that his opponent's action X is dominated, then... ; Step 3: etc.; Step N: etc.)

Step 1: For Camilla, Lilac is strictly dominated by Mango
Step 2: For Rebecca, being rational, Ultramarine strictly dominates dark orange
Step 3: For Camilla, being rational, Red is being strictly dominated by mango
Step 4: Only $(Ultra\ Marine; Mango)$ is left, which is the Nash equilibrium

Robert and Camilla have lost their phones, and must simultaneously decide whether to meet at the Soccer game or at the Opera. The following matrix describes their payoffs.

		Camilla	
		Soccer	Opera
Robert	Soccer	3,1	0,0
	Opera	0,0	1,3

C How many Nash Equilibria in pure strategies exist in this game? Find it/them.

There's two Nash equilibrium in this game $NE = [(Soccer; Soccer); (Opera; Opera)]$

D Calculate the Mixed Strategies Nash Equilibrium.

Camilla chooses $P = Soccer\ 1 - P = Opera$

$$EU_R(S) = 3 * P + (1 - P) * 0 = 3P$$
$$EU_R(O) = 0 * P + (1 - P) * 1 = 1 - P$$

Camilla wants to choose such that $EU_R(S) = EU_R(O)$

$$EU_R(S) = EU_R(O)$$
$$3P = 1 - P$$
$$4P = 1$$
$$P = Soccer = \frac{1}{4}$$

Then

$$1 - p = Opera = 1 - \frac{1}{4} = \frac{3}{4}$$

Camilla should choose soccer 25% of the time and opera 75% of the time

Robert chooses $Q = Soccer, 1 - Q = Opera$

$$EU_C(S) = 1 * Q + (1 - Q) * 0 = Q$$
$$EU_C(O) = 0 * Q + (1 - Q) * 3 = 3 - 3Q$$

Robert wants to choose such that $EU_C(S) = EU_C(O)$

$$EU_C(S) = EU_C(O)$$
$$Q = 3 - 3Q$$
$$4Q = 3$$
$$Q = Soccer = \frac{3}{4}$$

Then

$$1 - Q = Opera = 1 - \frac{3}{4} = \frac{1}{4}$$

Robert should choose soccer 75% of the time and opera 25% of the time

The mixed strategy Nash equilibrium is $NE = \left[\left(\frac{1}{4}; \frac{3}{4}\right); \left(\frac{3}{4}; \frac{1}{4}\right)\right]$

E Consider a prisoner's dilemma that is repeatedly played by the same two players 7 times. Can cooperation be sustained in this repeated game? Why or why not?

Cooperation in a prisoner's dilemma is hard to maintain, as they know there is an end of the game. Even if they cooperate for 6 games, they both know they can choose to not cooperate in the last round without facing any consequences for doing so. To repeat the logic to first 6 games, they both have incentive to not cooperate in all games as there is no consequences for doing so. It is rational to choose the highest payoff for the individual – so no it cooperation cannot be sustained when they know the amount of rounds the game will have.

2019 exam solved

Question 1 – constant return to scale

Question 1.

A. Suppose a firm has the following production function $Q = L^{0.25} \cdot K^{0.25}$, where Q is output, L is labor, and K is capital. Explain what is meant by constant returns to scale. Does the firm operate under constant returns to scale?

Constant return to scale is when the amount of output one puts into a firm, generates the same output, If $F(ak, aL) = aF(K, L)$ best exemplified by doubling the input will double the output, which will be done to the production function above. The Cobb Douglas rule for returns to scale, is that $\alpha + \beta = 1$, is constant return to scale. For this function $\alpha + \beta < 1$, and it is therefore decreasing returns to scale.

B. A firm operates under perfect competition and has the following cost function $TC = Q^3 - 4Q^2 + 5Q$, where Q is the quantity produced. Derive the firm's supply curve. At what level of production does the firm minimize its marginal cost?

Under perfect competition, the supply curve is equal to the MC curve.

$$MC = \frac{\partial TC}{\partial Q} = 3Q^2 - 8Q + 5$$

To find the level of production that minimize MC, set $MC' = 0$ and solve for Q

$$MC' = 6Q - 8 = 0$$

$$6Q = 8$$

$$Q = \frac{8}{6} \approx 1{,}33$$

C. What is the optimal level of production for the firm in the long run (under perfect competition)? Derive the aggregate supply curve in the long run.

In the long run, the production is determined by minimum average cost, find AC

$$AC = \frac{TC}{Q} = Q^2 - 4Q + 5$$

To find optimal level in the long run, take AC'=0 and solve for Q

$$AC' = 2Q - 4 = 0$$
$$Q^* = 2$$

The supply curve in the long run is the price, insert Q into AC

$$P^* = AC(2) = (2)^2 - 4(2) + 5 = 4 - 8 + 5 = 1$$

Since there is free entry under perfect competition, the aggregate supply curve is horizontal at a price of $P = 1$

D. Assume that aggregate demand in this market is given by Q = 15 - P. How many firms operate in the market in the long run under perfect competition? What is the value of producer and consumer surplus in this market?

To find aggregate demand, insert P^* in the aggregate demand function.

$$Q(P = 1) = 15 - 1 = 14$$

As the optimal production is $Q^* = 2$, divide aggregate demand by optimal production

$$Number\ of\ firms = \frac{Aggregate\ demand}{optimal\ production} = \frac{14}{2} = 7$$

There are 7 firms in the market in the long run

Consumer surplus is found by

The inverse demand function $P = 15 - Q$

$$CS = \frac{\Delta P * \Delta Q\ (aggregate\ demand)}{2} = \frac{14 * 14}{2} = 98$$

As there is no profit in the long run, the producer surplus is 0

$$TR - TC = 2 - 2 = 0$$

E. Suppose now that the government introduces a tax of 2 kroner on each unit of output in this market and levies the tax on consumers. What is the deadweight loss of introducing the tax?

Find new demand curve with tax

$$P + 2 = 15 - Q$$

Find inverse

$$Q = 13 - P$$
$$P = 13 - Q$$

Set equal to supply, $P = 1$

$$13 - Q = 1$$
$$Q_{tax} = 12$$

$$DWL = \frac{\Delta P * \Delta Q}{2} = \frac{tax * \Delta Q}{2} = \frac{2 * (14 - 12)}{2} = \frac{4}{2} = 2$$

The general formula is

$$DWL = \frac{(P_2 - P_1) * (Q_2 - Q_1)}{2}$$

Question 2.

Consider a monopolist with the following cost function: TC = 10Q, where Q is the quantity produced. The monopolist maximizes profit and is interested in selling the product in Denmark and the United Kingdom.

A. Assume that the monopolist only sells the product in Denmark, and that the inverse demand function in Denmark is $P_{DK} = 50 - 5Q_{DK}$. What is the optimal price, quantity produced and profit for the monopolist?

The monopolist has the following cost function $TC = 10Q$
The inverse demand function in Denmark: $P_{DK} = 50 - 5Q_{dk}$

Question A – Optimal quantity, price, and profit for the monopolist in Denmark

To find the optimal quantity, find MR and MC

$$MC = \frac{\partial TC}{\partial Q} = 10$$

In the monopoly, the MR is given by the same constant + double the coefficient

$$MR_{DK} = 50 - 10Q_{dk}$$

Set $MR = MC$ and solve for Q

$$MC = MR$$
$$50 - 10Q_{dk} = 10$$
$$10Q_{DK} = 40$$
$$Q_{DK} = 4$$

<u>The optimal quantity for the monopolist is $Q^* = 4$</u>

To find the price, substitute Q into the Cost function P_{DK}

$$P_{DK}(4) = 50 - 5(4) = 30$$

<u>The price is 30 at a quantity of 4</u>

To find the profit π substitute the above given numbers into the formula

$$\pi = p * q - TC$$
$$\pi = 30 * 4 - 10(4) = 120 - 40 = 80$$

<u>The profit for the monopolist is 80</u>

Question B – Optimal quantity, price, and profit for the monopolist in the United Kingdom

The inverse demand function in the UK: $P_{UK} = 100 - 5Q_{UK}$

As a function of P: $Q_{UK} = 20 - \frac{1}{5}P_{UK}$

To find the optimal quantity, find MR and MC

$$MC = \frac{\partial TC}{\partial Q} = 10$$

In the monopoly, the MR is given by the same constant + double the variable

$$MR_S = 100 - 10Q_{UK}$$

Set $MR = MC$ and solve for Q

$$MC = MR$$
$$100 - 10Q_{UK} = 10$$
$$10Q_{UK} = 90$$
$$Q_{UK} = 9$$

<u>The optimal quantity for the monopolist is $Q^* = 9$</u>

To find the price, substitute Q into the Cost function P_{UK}

$$P_S(9) = 100 - 5(9) = 55$$

<u>The price is 55 at a quantity of 9.</u>

To find the profit π substitute the above given numbers into the formula

$$\pi = p * q - TC$$
$$\pi = 55 * 9 - 10(9) = 495 - 90 = 405$$

<u>The profit for the monopolist is 405 on the british market.</u>

Question C – Perfect price discrimination - Optimal quantity, production, and profit for the monopolist

The optimal quantity and price for the monopolist when they can perfectly price discriminate is the same as in question A and B.

<u>So, for Denmark, the price is 30 at a quantity of 4</u>

<u>For the British market, it would be an optimal production of 9 with a price of 55</u>

To find the profit when the monopolist can price discriminate the two profits can be added together

$$\pi_{total} = \pi_{DK} + \pi_{UK} = 80 + 405 = 485$$

<u>The profit on the aggregate market is 485</u>

D. Derive social welfare in the two markets (Denmark and United Kingdom) under third degree price discrimination considered in Question C. Is social welfare higher in Denmark or the United Kingdom?

Since no fixed cost, the producer welfare is the profit

$$PS_{dk} = \pi = 80$$

$$CS_{dk} = \frac{\Delta P * \Delta Q}{2} = \frac{(50-30)*4}{2} = \frac{80}{2} = 40$$

$$TW = 80 + 40 = 120$$

$$PS_{UK} = \pi = 405$$

$$CS_{uk} = \frac{\Delta P * \Delta Q}{2} = \frac{(100-55)*9}{2} = \frac{405}{2} = 202{,}5$$

$$TW = 405 + 202{,}5 = 607{,}5$$

The social welfare is higher in the UK than in Denmark as $\mathbf{120 < 607{,}5}$

E. Assume that the monopolist operates in both Denmark and the United Kingdom but is not able to price discriminate between the two markets. What is the optimal price, quantity sold and profit?

When the monopolist is not able to price discriminate, the two markets should be considered as one.

Find the aggregate demand, by inverting the inverse demand functions

The inverse demand function in Denmark: $P_{DK} = 50 - 5Q_{dk}$
As a function of Q: $Q_{Dk} = 10 - 0{,}2P_{DK}$

The inverse demand function in the UK: $P_{UK} = 100 - 5Q_{UK}$
As a function of Q: $Q_{UK} = 20 - 0{,}2P_{UK}$

$$Q = Q_{DK} + Q_{UK}$$
$$Q = 10 - 0{,}2P_{DK} + 20 - 0{,}2P_{UK}$$
$$Q = 30 - 0{,}4P$$

Which get inversed again

$$0{,}4P = 30 - Q$$
$$P = 75 - 2{,}5Q$$

Find MR

$$MR = 75 - 5Q$$

Find optimal production by setting $MR = MC$

$$75 - 5Q = 10$$
$$65 = 5Q$$
$$Q = 13$$

The optimal production in the market is $\boldsymbol{Q^* = 13}$

To find the price at the given quantity, substitute $\boldsymbol{Q^* = 13}$ into the price function

$$P(Q = 13) = P = 75 - 2{,}5(13) = 75 - 32{,}5 = 42{,}5$$

<u>**The price is 42,5 at a quantity of 13**</u>

To find the profit π substitute the above given numbers into the formula

$$\pi = p * q - TC$$
$$\pi = 42{,}5 * 13 - 10(13) = 552{,}5 - 130 = 422{,}5$$

The profit when the monopolist cannot price discriminate is $\boldsymbol{422{,}5}$

Question 3.

A. Assume David has the following utility function over burgers (B) and slices of pizza (S): U(B, S) = 3B + S. What is the elasticity of substitution between burgers and slices of pizza?

Find MRS

$$MRS = -\frac{MU_B}{MU_S} = -\frac{3}{1} = -3$$

As MRS is a constant, the elasticity of substitution is infinite

B. Assume that the price of a burger is P_B = 60 kroner, and the price of a slice of pizza is P_S = 30 kroner. Suppose David has a budget of 120 kroner, what is his optimal demand for burgers and slices of pizza?

Finding the elasticity of substitution

$$P_B = 60, P_S = 30, M = 120$$

Find MRT

$$MRT = -\frac{P_B}{P_S} = -\frac{60}{30} = -2$$

As it is perfect substitutes, $MRS \neq MRT$, it will therefore be a corner value. As $MRS > MRT$, David will only consume 2 burgers

C. Assume Sophie has the following utility function over burgers (B) and slices of pizza (S): $U(B, S) = B^{2/3} \cdot S^{1/3}$. The price of a burger and a slice of pizza is the same as before (P_B = 60 kr. and P_S = 30 kr.), but Sophie has an unknown budget M for food. What is Sophie's optimal demand burgers and slices of pizza as a function of her budget, M?

$$U(B,S) = B^{\frac{2}{3}} * S^{\frac{1}{3}}$$

As it is a Cobb Douglas function, she spends 2/3 of M on Burgers and 1/3 of M on Pizza slices

$$P_B * Q_B = \frac{2}{3}M$$

$$60Q_B = \frac{2}{3}M$$

$$M = 90Q_B$$

$$Q_B = \frac{M}{90}$$

And

$$P_S * Q_S = \frac{1}{3}M$$

$$30Q_S = \frac{1}{3}M$$

$$90Q_s = M$$

$$Q_B = \frac{M}{90}$$

D. Anne has the following utility function over income: $U(Y) = Y^{0.5}$. Anne considers placing a bet on a lottery that pays 10,000 kroner with 1 percent probability, 100 kroner with 10 percent probability, and nothing otherwise. How much is Anne willing to pay for this bet? What is Anne's risk premium on this bet?

$$EV = 0{,}01(10.000) + 0{,}1(100) + 0{,}89(0) = 110$$

$$EU = 0{,}01(10.000)^{0{,}5} + 0{,}1(100)^{0{,}5} + 0{,}89(0)^{0{,}5} = 2$$

$$CE = EU^{-1} = EU^2 = 2^2 = 4$$

She is willing to pay 4 kroner for this bet

$$Risk\ premium = EV - CE = 106$$

Her risk premium is 106 kr.

E. Suppose the two prizes are now quadrupled such that the lottery now pays 40,000 kroner with 1 percent probability and 400 kroner with 10 percent probability, and nothing otherwise. What is the mean and standard deviation of the lottery? How much more is Anne willing to pay for this lottery compared to the lottery in Question 3D?

$$EV = 0{,}01(40000) + 0{,}1(400) = 440$$

$$Var = 0{,}01(40.000 - 440)^2 + 0{,}1(400 - 440)^2 + 0{,}89(0 - 440)^2$$
$$= 15.649.936 + 160 + 172.304 = 15.822.400$$

$$\sigma = \sqrt{Var} = 3977{,}74$$

How much is she willing to pay?

$$EU = 0{,}01(40000)^{0,5} + 0{,}1(400)^{0,5} = 2 + 2 = 4$$
$$CE = EU^{-1} = 4^2 = 16$$

She is willing to pay 16 kroner for the new lottery ticket

2018 Exam solved

Question 1.

A. Consider the following two utility functions:

(i) U = 10X + 5Y

(ii) $U = X^{0.25} \cdot Y^{0.75}$

What is the marginal rate of substitution between X and Y for each of the two utility functions?

i) $MRS = -\frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}} = -\frac{10}{5} = -2$

ii) $MRS = -\frac{\frac{\partial U}{\partial X}}{\frac{\partial U}{\partial Y}} = -\frac{0,25X^{-0,75}*Y^{0,75}}{0,75*X^{0,25}*Y^{-0,25}} = -\frac{0,25}{0,75}*\frac{Y}{X} = -\frac{1}{3}*\frac{Y}{X} = -\frac{Y}{3X}$

B. Assume that prices are given by $P_X = 10$ and $P_Y = 5$, and income is M = 100. What is the optimal level of consumption for each of the two utility functions in Part A?

$$MRT = -\frac{P_X}{P_Y} = -\frac{10}{5} = -2$$

i) As $MRS = MRT$, all corresponding bundles are optimal

ii) As it is a Cobb Douglas, it is seen that 25% is spent on good X and 75% on good Y. As the budget is M=100

$$10x = 0,25 * 100$$
$$10x = 25$$
$$x = 2,5$$

$$5Y = 0,75 * 100$$
$$5Y = 75$$
$$Y = 15$$

The optimal consumption is 2,5 of X and 15 of Y

C. Assume that you have a market where aggregate demand is given by $Q_D = 1{,}000 - 10P$ and aggregate supply is given by $Q_S = 10P$. What is the consumer and producer surplus in this market?

Find equilibrium $Q_D = Q_S$

$$1000 - 10P = 10P$$
$$1000 = 20P$$
$$P = 50$$

Insert $P^* = 50$ into any function

$$Q_S(P = 50) = 10(50) = 500$$

The quantity $Q^* = 500$

The consumer surplus

$$CS = \frac{\Delta P * \Delta Q}{2} = \frac{(100 - 50) * 500}{2} = 12.500$$
$$PS = \frac{\Delta P * \Delta Q}{2} = \frac{50 * 500}{2} = 12.500$$

D. Derive the price elasticity of demand, ε_D, and supply, ε_S, at the equilibrium price and quantity in Part C.

$$\epsilon_d = -10 * \frac{50}{500} = -1$$

$$\epsilon_s = 10 * \frac{50}{500} = 1$$

E. Let prices be in kroner and assume that the government levies a tax of 10 kroner per unit sold in this market. What is the effect on social welfare of introducing the tax?

Inverse demand curve

$$P_D = 100 - 0{,}1Q$$
$$P + 10 = 100 - 0{,}1Q$$
$$P = 90 - 0{,}1Q$$

Finding the initial demand curve

$$0{,}1Q = 90 - P$$
$$Q = 900 - 10P$$

Find equilibrium

$$Q_d = Q_S$$
$$900 - 10P = 10P$$
$$20P = 900$$
$$P^* = 45\ dkk$$

Insert P in demand curve to find Q

$$Q_S(P^* = 45) = 10(450) = 450$$

$$CS = \frac{\Delta P * \Delta Q}{2} = \frac{(100 - 45(+tax = 10)) * 450}{2} = \frac{45 * 450}{2} = 10.125$$

$$PS = \frac{\Delta P * \Delta Q}{2} = \frac{(45 - 0) * 450}{2} = 10.125$$

$$tax\ revenue = tax * Q = 4500\ DKK$$

Society welfare

$$SW = TS - TS_{Tax} = 25000 - 20250 = 4750\ dkk$$

Deadweight loss

$$DWL = SW - TR = 250\ dkk$$

Question 2.

A. A firm operates under perfect competition and has the following cost function:

$TC = 5Q^2 + 10Q + 5$, where Q is the quantity produced.

Derive (i) the variable cost, (ii) the average cost, (iii) the marginal cost, and (iv) the average variable cost.

i) $VC = 5Q^2 + 10Q$

ii) $AC = \frac{TC}{Q} = 5Q + 10 + \frac{5}{Q}$

iii) $MC = \frac{\partial TC}{\partial Q} = 10Q + 10$

iv) $AVC = \frac{VC}{Q} = 5Q + 10$

B. What is the optimal level of production for the firm in the long run (under perfect competition)? Derive the aggregate supply curve in the long run.

In the long run

$$AC' = 0$$

$$AC' = 5Q + 5 * Q^{-1} => 5 + 5 * (-1Q^{-1-1}) = 5 - \frac{5}{Q^2}$$

$$5 - \frac{5}{Q^2} = 0$$

$$5 = 5/Q^2$$

$$5Q^2 = 5$$

$$Q^2 = 1$$

$$Q^* = 1$$

The price in the long run is the average cost

$$P = AC(1) = 5(1) + 10 + \frac{5}{1} = 20$$

The supply curve in the long run is horizontal at $P = 20$

Assume that aggregate demand in this market is given by Q = 100 − P. How many firms operate in the market in the long run under perfect competition? What is the value of consumer surplus in this market?

The inverse demand is given at $P = 100 - Q$
Set equal to supply $P = 20$

$$100 - Q = 20$$

$$Q^*_{aggregate} = 80$$

As every firm produce 1, the number of firms in the market is 80

Consumer surplus

$$CS = \frac{(100 - 20) * 80}{2} = 3200$$

D. Assume now that there is only a single firm operating in this market. What is the optimal level of production and profit for the monopolist? What is the value of consumer surplus in this market?

$$MC = 10Q + 10$$
$$MR = 100 - 2Q$$
$$MR = MC$$
$$10Q + 10 = 100 - 2Q$$
$$12Q = 90$$
$$Q = 7{,}5$$

$$P(7{,}5) = 100 - 7{,}5 = 92{,}5$$

$$\pi = p * q - TC$$
$$92{,}5 * 7{,}5 - (5(7{,}5)^2 + 10(7{,}5) + 5) = 332{,}5$$

$$CS = \frac{(100 - 92{,}5) * 7{,}5}{2} = 28{,}125$$

E. Suppose now that the government introduces a tax of 30 kroner on each unit of output in this market and levies the tax on consumers. What is the optimal level of production and profit for the monopolist? Derive the value of consumer surplus in this market and compare it to consumer welfare in the two previous markets in Part C and D.

New demand function

$$p + 30 = 100 - Q$$
$$p = 70 - Q$$

$$MR = 70 - 2Q$$
$$MC = \frac{\partial TC}{\partial Q} = 10Q + 10$$

$$MR = MC$$
$$70 - 2Q = 10Q + 10$$
$$12Q = 60$$
$$Q^* = 5$$
$$P_d(5) = 70 - 5 = 65 = P^*$$

$$\pi = p * q - TC$$
$$\pi = 65 * 5 - 5(5)^2 + 10(5) + 5 = 325 - 180 = 145$$

$$CS = \frac{\Delta p + tax * \Delta Q}{2} = \frac{(100 - (65 + 30)) * 5}{2} = \frac{25}{2} = 12{,}5$$

Consumer welfares decrease when there's only one firm in the market compared to perfect competition and further reduced when a tax is imposed, that it is making the product relatively more expensive for the consumer.

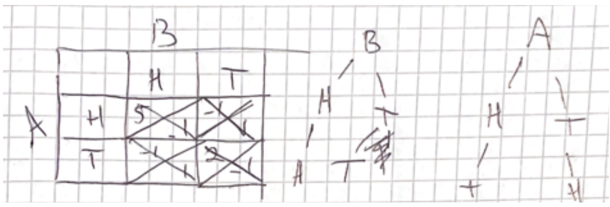
Question 3

A. Suppose Alice and Boris each have a coin, and they can choose to show either Heads or Tails. They simultaneously choose either Heads or Tails, and Table 1 below shows the payoffs to Alice and Boris from each strategy. What is the optimal strategy for Alice and Boris? Does a Nash equilibrium exist in this game?

Table 1: Payoff matrix (note: the first and second outcome in each cell refers to Alice and Boris, respectively)

		Boris	
		Heads	**Tails**
Alice	**Heads**	(5, −1)	(−1, 1)
	Tails	(−1, 1)	(2, −1)

No Pure strategy Nash equilibrium



(see the guide at the bottom for explanation)

B. Suppose that Boris flips the coin and randomly selects Heads or Tails with a 50-50 probability. What is the optimal mixed strategy for Alice?

If Alice chooses heads the expected payoff is

$$EP(H) = 0{,}5(5) + 0{,}5(-1) = 2$$

If Alice chooses tails the expected payoff

$$EP(T) = 0{,}5(-1) + 0{,}5(2) = 0{,}5$$

Then she should choose like this:

$$p * EV(H) = (1-p) * EV(T)$$
$$p * 2 = (1-p) * 0{,}5$$
$$2p = 0{,}5 - 0{,}5p$$
$$2{,}5p = 0{,}5$$
$$\boldsymbol{p = 0{,}2}$$

Mixed strategy Nash equilibrium: $(\frac{1}{5};\frac{4}{5})$

C. Eric considers buying a lottery ticket from Frank that gives a 50-50 probability of winning 25 kroner or 100 kroner. Assume Eric's utility function over income is given by $U_E(Y) = Y^{0.5}$, and Franks utility function is given by $U_F(Y) = Y$. How much is Eric willing to pay for the lottery ticket? How much is Frank willing to accept for the lottery ticket?

$$EU_E = 0{,}5\sqrt{25} + 0{,}5\sqrt{100} = 7{,}5$$
$$WTP_E = CE = EU^{-1} = 7{,}5^2 = 56{,}25$$
$$EU_F = 0{,}5(25) + 0{,}5(100) = 62{,}5$$
$$WTP_F = CE = EU^{-1} = 62{,}5$$

As $WTP_E < ETP_F$, Frank is not selling his ticket

D. Suppose Eric's utility function is the same as in Part C and Frank's utility function now is $U_F(Y) = Y^{0.25}$. Will Eric and Frank be willing to trade the lottery ticket? Explain your answer.

$$EU_E = 0{,}5\sqrt{25} + 0{,}5\sqrt{100} = 7{,}5$$
$$WTP_E = CE = EU^{-1} = 7{,}5^2 = 56{,}25$$
$$EU_F = 0{,}5(25)^{0{,}25} + 0{,}5(100)^{0{,}25} = 2{,}7$$
$$WTP_F = CE = EU^{-1} = 2{,}7^4 = 53{,}14$$
$$WTP_E > WTP_F$$

Frank will sell as he is more risk averse than Eric

E. Eric now considers buying a lottery ticket from Frank that gives a 50% probability of winning 25 kroner and a 50% probability of winning a new lottery ticket that gives a 10% probability of winning 10,000 kroner. Assume Eric's utility function over income is $U_E(Y) = Y^{0.5}$. How much is Eric willing to pay for the lottery ticket? What is Eric's risk premium in kroner for this lottery ticket?

$$EV = 0{,}5(25) + 0{,}5 * 0{,}1(10000) = 512{,}5\ kr.$$
$$EU = 0{,}5(25)^{0{,}5} + 0{,}5 * 0{,}1(10000)^{0{,}5} = 7{,}5$$
$$WTP = CE = EU^{-1} = 7{,}5^2 = 56{,}25\ kr.$$
$$RP = EV - CE = 456{,}25\ kr.$$

2017 Exam - solved

Question 1.

A. Suppose a consumer has the following utility function $U = X^2 \cdot Y$. Assume that prices are given by $P_X = 1$ and $P_Y = 2$, and income is M = 150. What is the optimal level of consumption and utility for the consumer?

$$U = X^2 * Y$$
$$MRS = \frac{2X * Y}{x^2} = \frac{2 * X * Y}{X * X} = \frac{2Y}{X}$$
$$MRT = \frac{1}{2}$$
$$MRS = MRT$$
$$\frac{2Y}{X} = \frac{1}{2}$$
$$2Y = \frac{1}{2}X$$
$$4Y = X$$
$$Y = 0{,}25X$$

$$M = P_x * X + P_Y * Y$$
$$150 = 1X + 2(0{,}25X)$$
$$150 = 1{,}5X$$
$$X^* = 100$$

$$Y^* = 0{,}25(100) = 25$$

Insert Y and X in the utility function

$$U(100{,}25) = 100^2 * 25 = 250.000$$

B. A consumer has the following inverse demand function: P = 100 / Q, where P is price and Q is quantity.
Derive the price elasticity of demand, ε. What is the price elasticity of demand when P = 1 and Q = 100?

$$P = \frac{100}{Q}$$
$$Q = \frac{100}{P} = 100 * P^{-1}$$
$$\varepsilon = \frac{\partial Q}{\partial P} * \frac{Q}{P}$$
$$\frac{\partial Q}{\partial P} = 100 * -1P^{-1} = -\frac{100}{p^2}$$

$$-\frac{100}{p^2} * \frac{P}{Q} = \frac{100P}{P^2 * Q} = -\frac{100}{P * Q}$$

Insert values

$$\varepsilon = -\frac{100}{100} = -1$$

C. Eric has the following utility function over income: U(Y) = Y^r, where Y is income and r is a parameter that measures risk aversion. Derive the first- and second order derivatives with respect to income Y for this utility function. For what values of r is Eric risk neutral, risk averse and risk seeking?

$$U(Y) = Y^r$$
$$U(Y)' = rY^{r-1}$$
$$U(Y)'' = r * (r-1)Y^{r-2}$$

Eric is risk neutral at $r = 1$
Eric is risk averse at $r = [0 < r > 1]$
Eric is risk seeking at $r > 1$

D. Assume that r = 0.5 such that Eric has the following utility function over income: U(Y) = $Y^{0.5}$. How much is Eric willing to pay for a lottery that pays 100 with 20 percent probability and 25 with 80 percent probability? What is Eric's risk premium for this lottery?

$$EV = 0{,}2(100) * 0{,}8(25) = 20 + 20 = 40$$
$$EU = 0{,}2(100)^{0{,}5} + 0{,}8(25)^{0{,}5} = 2 + 4 = 6$$
$$WTP = CE = EU^{-1} = EU^2 = 6^2 = 36$$
$$RP = EV - CE = 40 - 36 = 4$$

E. How much is Eric willing to pay for a lottery that now pays 10,000 with 20 percent probability and 2,500 with 80 percent probability? What is Eric's risk premium for this lottery? How much higher is the risk premium for this lottery compared to the lottery in Question D.

$$EV = 0{,}2(10000) + 0{,}8(2500) = 4000$$
$$EU = 0{,}2(10000)^{0{,}5} + 0{,}8(2500)^{0.5} = 60$$
$$WTP = CE = EU^{-1} = EU^2 = 60^2 = 3600$$
$$\underline{RP = EV - CE = 4000 - 3600 = 400}$$

Question 2.

A. A firm operates under perfect competition and has the following total cost function: $TC = Q^2 + 2Q + 1$, where Q is the quantity produced.
Assume that the market price under perfect competition is P = 10. What is the optimal level of production for the firm at this market price in the short run?

$$MR = P = 10$$
$$MC = \frac{\partial TC}{Q} = 2Q + 2$$
$$MR = MC$$
$$10 = 2Q + 2$$
$$Q = 4$$

B. What is the optimal level of production for the same firm in the long run?

$$AC = \frac{TC}{Q} = Q + 2 + \frac{1}{Q}$$
$$AC' = 0$$
$$AC' = 1 + 1 * -1 * Q^{-2}$$
$$AC' = 1 - \frac{1}{Q^2} = 0$$
$$1 - \frac{1}{Q^2} = 0$$
$$\frac{1}{Q^2} = 1$$
$$Q^2 = 1$$
$$Q = 1$$

(if to find price=supply curve, insert Q into AC)

C. Suppose that all firms in the market have the same cost function as above and aggregate demand is given by Q = 20 – 2P. How many firms operate in the market in the long run under perfect competition?

$$Q = 20 - 2P$$

Find price

$$AC = \frac{TC}{Q} = Q + 2 + \frac{1}{Q}$$
$$AC(1) = 1 + 2 + 1 = 4$$

Price is 4

$$Q(P = 4) = 20 - 2(4) = 12$$

The total demand is 12

$$\text{number of firms} = \frac{Q_{total}}{Q_{individual}} = \frac{12}{1} = 12$$

D. What is the optimal level of production for a monopolist with the same cost function ($TC = Q^2 + 2Q + 1$) and aggregate demand (Q = 20 – 2P) as above? What is the profit for the monopolist?

$$P = 10 - 0{,}5Q$$

$$MC = 2Q + 2$$
$$MR = 10 - Q$$
$$MC = MR$$
$$2Q + 2 = 10 - Q$$
$$3Q = 8$$
$$Q^* = \frac{8}{3}$$

$$P(3) = 10 - 0{,}5 * 2{,}67 = 8{,}67$$
$$\pi = P * Q - TR$$
$$\pi = 8{,}67 * 2{,}67 - (2{,}67^2) + 2(2{,}67) + 1 = 23{,}15 - 13{,}47 = 9{,}68$$

E. Suppose now that the government introduces a tax of 2 kroner on each unit of output sold in the market and levies the tax on consumers. What is the optimal level of production and profit for the monopolist? (Use the same cost function and aggregate demand function as the previous question)

$$P + 2 = 10 - 0,5Q$$
$$P = 8 - 0,5Q$$
$$MR = 8 - Q$$
$$MC = 2Q + 2$$
$$MR = MC$$
$$8 - Q = 2Q + 2$$
$$3Q = 6$$
$$Q^* = 2$$

$$P(Q = 2) = 8 - 0,5(2) = 7 = P^*$$

$$\pi = P * Q - TR$$
$$\pi = 7 * 2 - (2)^2 + 2(2) + 1 = 14 - 9 = 5$$

Question 3

A. A firm operates under perfect competition and has the following production function: $Q = K^{0.25} \cdot L^{0.75}$, where Q is the quantity produced, L is labor and K is capital. What are the marginal products of capital and labor?

$$MP_K = 0,25 * K^{-0,75} * L^{0,75}$$
$$MP_L = 0,75 * K^{0,25} * L^{-0,25}$$

B. Let the price of capital and labor be P_K and P_L, respectively. Suppose that the firm produces Q = 100 units and sells the output for 10 kroner per unit. What is the optimal demand for capital and labor by the firm at that output level and price per unit?

$$MRTS = \frac{MP_L}{MP_K} = \frac{0,75 * K^{0,25} * L^{-0,25}}{0,25 * K^{-0,75} * L^{0,75}} = \frac{0,75}{0,25} * \frac{K}{L} = \frac{3K}{L}$$
$$\frac{3K}{L} = \frac{P_L}{P_K}$$
$$3K * P_K = P_L * L$$
$$P * Q = P_K * K + P_L * L$$
$$1000 = 3 * K * P_K + K * P_K$$
$$1000 = 4 * K * P_K$$
$$\frac{250}{P_K} = K$$

$$P * Q = P_K * \frac{250}{P_K} + P_L * L$$
$$1000 = 250 + P_L * L$$
$$750 = P_L * L$$
$$L = \frac{750}{P_L}$$

C. Let the price of capital be $P_K = 5$ and $P_L = 10$. What is the optimal demand for capital and labor for the firm? (Note: use the same production function, output level and output price as before)

$$\frac{250}{P_K} = K$$
$$\frac{250}{5} = K$$
$$K^* = 50$$
$$L = \frac{750}{10}$$
$$L^* = 75$$

D. Assume now that we have a market in which aggregate demand is given by $Q_D = 120 - 10P$ and aggregate supply is given by $Q_S = 2P$. Derive the market equilibrium. What is the producer surplus and consumer surplus in this market?

$$120 - 10P = 2P$$
$$P = 10$$

$$Q_S(10) = 2(10) = 20$$

$$P = 12 - 0{,}1Q$$

$$CS = \frac{\Delta P * \Delta Q}{2} = \frac{(12 - 10) * 20}{2} = \frac{40}{2} = 20$$

$$PS = \frac{\Delta P * \Delta Q}{2} = \frac{10 * 20}{2} = 100$$

E. Suppose that the government levies a tax of 3 kroner per unit sold in the market and levies the tax on producers. What are the new prices (including and excluding the tax) and the new quantity sold in equilibrium? What is the welfare cost to consumers and producers of imposing the tax?

Inverse demand to initial demand

$$Q_s = 2P$$
$$P = \frac{Q_S}{2}$$

Impose tax

$$P - 3 = \frac{Q_S}{2}$$
$$P = 0{,}5Q_S + 3$$

Inverse demand

$$P - 3 = 0{,}5Q_S$$
$$2P - 6 = Q_{S^*_{tax}}$$

$$Q_D = Q_{s_{tax}}$$
$$120 - 10P = 2P - 6$$
$$126 = 12P$$
$$\boldsymbol{P_c = 10{,}5}$$

$$\boldsymbol{P_P = P - tax = 10{,}5 - 3 = 7{,}5}$$

$$\boldsymbol{Q(P = 10{,}5) = 2(10{,}5) - 6 = 15 = Q^*}$$

$$Height = P_C - P_P = 10{,}5 - 7{,}5 = 3 = tax\ value$$
$$Base = Q_{old} - Q_{tax} = 20 - 15 = 5$$

$$DWL = \frac{height * base}{2} = \frac{(P_C - P_P) * (Q_{old} - Q_{tax})}{2} = \frac{3 * 5}{2} = \mathbf{7{,}5}$$

2016 Exam - solved

Question 1.

A. A firm operates under perfect competition and has the following production function: $Q = K^{0.5} \cdot L^{0.5}$, where Q is the quantity produced, L is labor and K is capital. What is the marginal rate of technical substitution?

$$Q = L^{0,5} * K^{0,5}$$

$$MRTS = -\frac{MP_L}{MP_K} = \frac{0,5 * K^{0.5} * L^{-0,5}}{0,5 * K^{-0,5} * L^{0,5}} = -\frac{K}{L}$$

B. Suppose that the capital stock is fixed in the short term and equal to 9 and the price of labor is w = 9. What is the variable cost function in the short run for a firm with the same production function as in question 1A?

$$\overline{\overline{K}} = 9, w = 9$$

$$VC = w * L$$
$$Q_{SR} = L^{0,5} * (9)^{0,5}$$
$$Q_{SR} = L^{0,5} * 3$$
$$\sqrt{L} = \frac{Q_{SR}}{3}$$
$$L = \frac{Q_{SR}^2}{9}$$
$$VC(Q) = 9 * \frac{Q^2}{9} = Q^2$$

C. Derive the supply curve in the short term for the firm given the production function, capital stock and prices given earlier. Suppose there are 10 firms operating in the market in the short term with the same technology as specified above, what is the aggregate supply function in the market?

MC= supply

$$MC = \frac{\partial TC}{\partial Q} = 2Q$$
$$MC = P = 2Q$$
$$Q = \frac{P}{2}$$
$$AS = 10 * \frac{P}{2} = 5P$$

D. Assume that aggregate demand in the short run is given by $Q_D = 100 - 5P$ and aggregate supply is given by $Q_S = 5P$. Derive the market equilibrium in the short run. What is the producer surplus and consumer surplus in this market?

$$Q_D = 100 - 5P$$
$$Q_S = 5P$$
$$Q_D = Q_S$$
$$100 - 5P = 5P$$
$$P = 10$$

$$Q_S(10) = 5(10) = 50$$

$$Q^* = 50$$
$$P^* = 10$$

Consumer surplus

$$CS = \frac{\Delta P * \Delta Q}{2} = \frac{(20 - 10) * 50}{2} = 250$$

Producer surplus

$$PS = \frac{(10 - 0) * 50}{2} = 250$$

E. Suppose that the government levies a tax of 4 per unit sold in the market, what is the welfare cost of imposing the tax? Does it matter to the results whether the tax is levied on consumers or producers?

Assume $tax = 4$, does it matter whether it's on producers or consumers?

No, it does not, but here is why:

Supply

$$P = \frac{Q}{5}$$

Demand

$$P + 4 = 20 - \frac{Q}{5}$$
$$P = 16 - 0{,}2Q$$

$$0{,}2Q = 16 - 0{,}2Q$$
$$0{,}4Q = 16$$
$$Q^* = 40$$
$$P_P(40) = 16 - 0{,}2(40) = 8 = P^*$$

$$P_C = 8 + t = 8 + 4 = 12$$

Same for if added to producer

Question 2

A. Lisa considers buying a lottery ticket that pays out 1,000 kroner with a 20% probability and 0 kroner with an 80% probability. Her utility function over income is specified as $U(Y) = Y^2$, where Y is income. What is the expected value and the expected utility of the lottery?

B. How much is Lisa willing to pay for the lottery ticket? What is the risk premium in this case?

$$U(Y) = Y^2$$
$$EU = 0{,}2 * 1000 = 200$$
$$EU = 0{,}2 * (1000)^2 = 200.000$$
$$CE = EU^{-1} = \sqrt{200.000} = 447{,}21$$
$$Risk\ premium = EV - CE = 200 - 447{,}21 = -247{,}21$$

C. Suppose Lisa buys the lottery ticket and wins the prize of 1,000 kroner. She decides to spend the entire amount on food and drinks and has the following utility function over the two types of goods: $U(F, D) = F^{0.75} D^{0.25}$. The price of food is $P_F = 150$ and the price of drinks is $P_D = 50$. What is her optimal demand for food and drinks?

$$1000 = 150F + 50D$$

As it is a Cobb Douglas, the exponents tell how much of the income is spend on either good $\frac{3}{4}$ is spent on Food and $\frac{1}{4}$ on drinks

$$150F = 1000 * 0{,}75$$
$$150F = 750$$
$$\boldsymbol{F = 5}$$

$$50D = 0{,}25 * 1000$$
$$50D = 250$$
$$\boldsymbol{D = 5}$$

D. Derive Lisa's demand curves for food and drinks. What is her price elasticity of demand for food and drinks at the prices given above ($P_F = 150$ and $P_D = 50$)?

$$Q = \frac{750}{P_F}$$
$$Q = \frac{250}{P_D}$$

$$\varepsilon = \frac{\Delta Q}{\Delta P} * \frac{P}{Q}$$

For ε_{food}

$$\frac{\Delta Q}{\Delta P} = \left(\frac{750}{P}\right)^1 = (750 * P^{-1})^1 = -750 * P^{-2} = -750 * \frac{1}{P^2} = -\frac{750}{P^2}$$
$$\varepsilon_{food} = -\frac{750}{150^2} * \frac{150}{5} = -\frac{112.500}{112500} = -1$$

For ε_{drinks}

$$\frac{\Delta Q}{\Delta P} = \left(\frac{250}{p}\right)^1 = (250 * p^{-1})^1 = -250 * p^{-2} = -\frac{250}{p^2}$$
$$\varepsilon_{drinks} = -\frac{250}{50^2} * \frac{50}{5} = \frac{12.500}{12.500} = -1$$

E. Now suppose Lisa's utility function is given by U(F, D) = min{F, 3D}. Assume that prices are the same as above (P_F = 150 and P_D = 50). Derive her demand for food and drinks, respectively.

$$U(X,Y) = \min\{X, 3Y\}$$

s.t.

$$1000 = 150X + 50Y$$

$$Y = \frac{1000}{50} - \frac{150}{50}x$$

$$Y = 20 - 3X$$

It is a linear function

The ratio is given, so no MRS

$$x = 3Y$$

Budget constraint

$$1000 = 150(3Y) + 50Y$$
$$1000 = 450Y + 50Y$$
$$\boldsymbol{Y = 2}$$

Insert in

$$x = 3Y$$

$$\boldsymbol{x = 3(2) = 6}$$

Question 3
Price discrimination – Denmark and Sweden (red are things you can change out to make it easy for yourself)

The monopolist has the following cost function $TC = 4Q$
The inverse demand function in Denmark: $P_{DK} = 160 - 2Q_{dk}$
The inverse demand function in Sweden: $P_S = 100 - Q_S$

Question A – Optimal quantity, price, and profit for the monopolist in Denmark

To find the optimal quantity, find MR and MC

$$MC = \frac{\partial TC}{\partial Q} = 4$$

In the monopoly, the MR is given by the same constant + double the variable

$$MR_{DK} = 160 - 4Q_{dk}$$

Set $MR = MC$ and solve for Q

$$MC = MR$$
$$160 - 4Q_{DK} = 4$$
$$4Q_{DK} = 156$$
$$Q_{DK} = 39$$

The optimal quantity for the monopolist is $Q^* = 39$

To find the price, substitute Q into the Cost function P_{DK}

$$P_{DK}(39) = 160 - 2(39) = 82$$

The price is 82 at a quantity of 39

To find the profit π substitute the above given numbers into the formula

$$\pi = p * q - TC$$
$$\pi = 82 * 39 - 4(39) = 3198 - 156 = 3042$$

The profit for the monopolist is 3042

Question B – Optimal quantity, price, and profit for the monopolist in Sweden

To find the optimal quantity, find MR and MC

$$MC = \frac{\partial TC}{\partial Q} = 4$$

In the monopoly, the MR is given by the same constant + double the variable

$$MR_S = 100 - 2Q_S$$

Set $MR = MC$ and solve for Q

$$MC = MR$$
$$100 - 2Q_S = 4$$
$$2Q_S = 96$$
$$Q_{DK} = 48$$

<u>The optimal quantity for the monopolist is $\boldsymbol{Q^* = 48}$</u>

To find the price, substitute Q into the Cost function P_S

$$P_S(48) = 100 - 48 = 52$$

<u>The price is 52 at a quantity of 48.</u>

To find the profit π substitute the above given numbers into the formula

$$\pi = p * q - TC$$
$$\pi = 52 * 48 - 4(48) = 2496 - 192 = 2304$$

<u>The profit for the monopolist is 2304 on the Swedish market.</u>

Question C – Perfect price discrimination - Optimal quantity, production, and profit for the monopolist

The optimal quantity and price for the monopolist when they can perfectly price discriminate is the same as in question A and B.

<u>So for Denmark it would be an optimal production of 39 with a price of 82</u>
<u>For Sweden it would be an optimal production of 48 with a price of 52</u>

To find the profit when the monopolist can price discriminate the two profits can be added together

$$\pi_{total} = \pi_{DK} + \pi_S = 3042 + 2304 = 5346$$

<u>The profit on the aggregate market is 5346</u>

Question D – No price discrimination

When the monopolist is not able to price discriminate, the two markets should be considered as one.

Find the aggregate demand, by inverting the inverse demand functions

The inverse demand function in Denmark: $P_{DK} = 160 - 2Q_{dk}$
As a function of P: $Q_{Dk} = 80 - 0{,}5P_{DK}$
The inverse demand function in Sweden: $P_S = 100 - Q_S$
As a function of P: $Q_S = 100 - P_S$

$$Q = Q_{DK} + Q_S$$
$$Q = 180 - 1{,}5P$$
$$P = 120 - \frac{2}{3}Q$$

Find MR

$$MR = 120 - \frac{4}{3}Q$$

Find optimal production by setting $MR = MC$

$$120 - \frac{4}{3}Q = 4$$
$$116 = \frac{4}{3}Q$$
$$Q = 87$$

The optimal production in the market is $\boldsymbol{Q^* = 87}$

To find the price at the given quantity, substitute $\boldsymbol{Q^* = 87}$ into the price function

$$P = 120 - \frac{2}{3}(87) = 120 - 58 = 62$$

<u>The price is 62 at a quantity of 87</u>

To find the profit π substitute the above given numbers into the formula

$$\pi = p * q - TC$$

$$\pi = 62 * 87 - 4(87) = 5394 - 348 = 5046$$

The profit when the monopolist cannot price discriminate is 5046.

Question E – Welfare loss 3D >< perfect competition

The Surplus for no price discrimination is as following

As there is no fixed cost, the profit is the producer surplus: $PS = 5046$

To find consumer surplus (Original price (from the demand function =120) and the price found(= 62))

$$CS = \frac{\Delta P * \Delta Q}{2} = \frac{(120 - 62) * 87}{2} = \frac{58 * 87}{2} = 2523$$

To find the welfare loss, find the production and price for perfect competition

$$MC = 4$$
$$MC = P$$
$$4 = 120 - \frac{2}{3}Q$$
$$\frac{2}{3}Q = 116$$
$$\boldsymbol{Q = 174}$$

Find price by inserting into the price function

$$P(174) = 120 - \frac{2}{3}(174) = 120 - 116 = 4$$

The price in perfect competition is 4

Find producer surplus (the price is MC= 4 and the price is 4)

$$PS = \frac{\Delta P * \Delta Q}{2} = \frac{(4 - 4) * 174}{2} = \frac{0}{2} = 0$$

Find Consumer surplus

$$CS = \frac{\Delta P * \Delta Q}{2} = \frac{(120 - 4) * 174}{2} = \frac{20.184}{2} = 10.092$$

The welfare loss is given by

$$Welfare\ loss = Surplus_{competetion} - Surplus_{monopoly}$$
$$\boldsymbol{WL = 10.092 - (5046 + 2523) = 2523}$$

<u>The welfare loss due to the monopoly is 2523</u>

2015 exam - solved

Question 1.

A. A firm operates under perfect competition and has the following cost function: $TC = 2Q^2 + 4Q + 2$, where Q is the quantity produced.

Derive (i) the average fixed cost, (ii) marginal cost, and (iii) average variable cost.

$$AC = \frac{TC}{Q} = 2Q + 4 + \frac{2}{Q}$$
$$MC = \frac{\partial TC}{\partial Q} = 4Q + 4$$
$$AVC = \frac{VC}{Q} = 2Q + 4$$

B. Assume that the market price under perfect competition is P = 10. What is the optimal level of production for the firm at this market price in the short and long run?

Short run

$$P = MC$$
$$10 = 4Q + 4$$
$$4Q = 6$$
$$Q^* = 1{,}5$$

Long run

$$AC' = 0$$
$$AC' = 2 - \frac{2}{Q^2}$$
$$2 - \frac{2}{Q^2} = 0$$
$$\frac{2}{Q^2} = 2$$
$$Q^2 = 1$$
$$Q^* = 1$$

C. Assume that all firms in the market have the same cost function as above and aggregate demand is given by Q = 40 − P. How many firms operate in the market in the long run under perfect competition?

$$AC(1) = 2(1) + 4 + \frac{2}{1} = 2 + 4 + 2 = 8 = P$$
$$Q_{aggregate} = 40 - P = 40 - 8 = 32$$

$$Number\ of\ firms = \frac{Q_{aggregate}}{Q_{individual}} = \frac{32}{1} = 32\ firms$$

D. What is the optimal level of production for a monopolist with the same cost function and aggregate demand as above? What is the profit for the monopolist?

$$Q = 40 - P$$
$$P = 40 - Q$$
$$MR = 40 - 2Q$$
$$MC = \frac{\partial TC}{\partial Q} = 4Q + 4$$

$$MR = MC$$
$$40 - 2Q = 4Q + 4$$
$$6Q = 36$$
$$Q^* = 6$$
$$P(6) = 40 - 6 = 34 = P^*$$

$$\pi = P * Q - TR$$
$$\boldsymbol{\pi = 34 * 6 - 2(6)^2 + 4(6) + 2 = 204 - 72 - 24 - 2 = 106}$$

E. Consider the following three production functions:

(i) Q = L + K

(ii) Q = L · K

(iii) $Q = L^{0.5} \cdot K^{0.5}$

Explain what is meant by constant returns to scale. Does any of the three production functions have constant returns to scale?

Constant return to

Question 2.

A. Consider the following three utility functions:

(i) U = X + Y

(ii) U = X · Y

(iii) U = min(X, Y)

What is the marginal rate of substitution between X and Y for each of the three utility functions?

i) $MRS = -\frac{1}{1} = -1$ – perfect substitutes

ii) $MRS = -\frac{Y}{X}$

iii) Is a perfect complement, so will only be consumed together, therefore it is 0.

B. Assume that prices are given by $P_X = 1$ and $P_Y = 2$, and income is M = 100. What is the optimal level of consumption for each of the three utility functions in part A?

$$MRT = \frac{P_X}{P_Y} = \frac{1}{2} = -0{,}5$$
$$100 = 1X + 2Y$$

$$MRS = MRT$$

For i)

$$-1 \neq -0{,}5$$
$$|MRS| > |MRT|$$

Only good X will be consumed

For ii)

$$-\frac{Y}{X} = -\frac{1}{2}$$
$$Y = 0{,}5X$$
$$100 = 1X + 2(0{,}5x)$$
$$100 = 2x$$
$$X^* = 50$$
$$Y^* = 0{,}5(50) = 25$$

For iii)

$$Y = X$$
$$100 = 1X + 2(X)$$
$$100 = 3X$$
$$X = \frac{100}{3}$$
$$Y = \frac{100}{3}$$

C. Assume that the price of Y falls and is now $P_Y = 1$. What is the optimal level of consumption for each of the three utility functions in part A?

$$MRT = -\frac{1}{1} = -1$$

For i)

$$MRS = MRT$$
$$-1 = -1$$

All bundles are optimal

For ii)

$$MRS = MRT$$

$$-\frac{Y}{X} = -1$$
$$Y = X$$
$$100 = 1X + 1(X)$$
$$\boldsymbol{X^* = 50}$$
$$\boldsymbol{Y^* = 50}$$

For iii)

$$Y = X$$
$$100 = 1X + 1(X)$$
$$\boldsymbol{X^* = 50}$$
$$\boldsymbol{Y^* = 50}$$

D. A consumer has the following inverse demand function: P = 20 – Q, where P is price and Q is quantity.
Derive the price elasticity of demand, ε. What is the price P when the price elasticity of demand is ε = –3?

$$\varepsilon = -1 * \frac{P}{Q}$$

Inverse demand

$$Q = 20 - P$$

Substitute it in the function

$$-3 = -\frac{P}{20 - P}$$
$$3 = \frac{P}{20 - P}$$
$$60 - 3P = P$$
$$4P = 60$$
$$P = 15$$

E. Assume that the supply function is given by 3Q = P – 10 and the inverse demand function is given by P = 20 – Q (same as in part D).
Derive the equilibrium price and quantity and calculate the consumer surplus in market equilibrium.

Inverse supply

$$3Q = P - 10$$
$$P = 10 + 3Q$$
$$20 - Q = 10 + 3Q$$
$$4Q = 10$$
$$Q^* = 2{,}5$$

$$P(Q=2{,}5)=20-2{,}5=17{,}5=P^*$$

Consumer surplus

$$CS=\frac{6{,}25}{2}=3{,}13$$

Question 3.

A. Consider the following three utility functions:

(i) U(W) = exp(W)

(ii) U(W) = ln(W)

(iii) U(W) = W

For each of the three utility functions assess whether a person with these preferences is risk averse, risk seeking or risk neutral.

i) The exponential function is convex; therefore, the person is risk seeking
ii) The logarithmic function is concave and therefore the person in risk averse
iii) The linear function implies that the person is risk neutral

B. David is considering buying a car insurance. His car is valued at 250,000 kroner and the probability is 10% of having an accident within the next year that writes off the value of his car. His annual income is 500,000 kroner and his utility function is given by $U(W) = W^{0.5}$. He can buy an annual comprehensive insurance for 25,000 kroner and is compensated in full if he has an accident that writes off the value of his car.
What is David's expected utility of buying and not buying the insurance, respectively?
Will he buy the insurance at a price of 25,000 kroner?

$$U(W)=\sqrt{W}$$
$$EV_{noinsurance}=0{,}9(500000)+0{,}1(250000)=450.000+25.000=475.000$$
Wealth if buying insurance $500.000+250.000=475.000$
$$U(W_{insurance})=475.000^{0{,}5}=689{,}2$$
$$U(EV_{no\ insurance})=0{,}9(500000)^{0{,}5}+0{,}1(250000)^{0{,}5}=686{,}3$$
$$U(W_{insurance})>U(EV_{no\ insurance})$$

David will buy the car insurance

C. How much is David willing to pay in annual premium for the car insurance?

$$CE = EU^{-1} = 686{,}4^2 = 471.144{,}96$$
$$WTP = W - CE = 500.000 - 686{,}4^2 = 28.855{,}04$$

D. What is the risk premium David is willing to pay in annual premium for the car insurance? (Hint: the risk premium is the difference between the willingness to pay for the car insurance and the expected value of the loss if he writes off the value of the car.)

$$RP = WTP - Insurance = 28.855{,}04 - 25000 = 3.855{,}04$$

E. Suppose David is optimistic and believes that he has a 1% chance of having an accident within the next year that writes off the value of his car. How much is David willing to pay in annual premium for the car insurance? Will he buy the insurance at a price of 2,500 kroner?

$$EV_{no\,insurance} = 0{,}99(500000) + 0{,}01(250000) = 497.500$$
$$EU_{no\ insurance} = 0{,}99(500000^{0,5}) + 0{,}01(250000^{0,5}) = 705{,}04$$
$$EU_{insurance} = 497.500^{0,5} = 705{,}34$$
$$EU_{no} < EU_{insurance}$$

He will buy the insurance

2014 Exam – Solved

1. Suppose there are only two consumers in the market for umbrellas. Consumer A and Consumer B. Their demand functions are $P = 20 - 4Q_A$ and $P = 20 - 2Q_B$ respectively.
   a) What is the market demand function for umbrellas?
   b) What is the price elasticity of demand in the market when the price is 4
   c) Suppose the supply function for this market is $P = 6 + Q$. What will be equilibrium price and quantity?

Invert both functions

$$Q_a = 5 - 0{,}25P$$
$$Q_b = 10 - 0{,}5P$$

$$\boldsymbol{Q = 15 - 0,75P}$$
$$\boldsymbol{P = 20 - \frac{4}{3}Q}$$

Insert P in the demand function

$$\boldsymbol{Q = 15 - 0,75 * 3 = 12}$$

$$\varepsilon = \frac{\Delta Q}{\Delta P} * \frac{P}{Q}$$

$$\frac{\Delta Q}{\Delta P} = -0{,}75 * \frac{4}{12}$$

$$\varepsilon = -\frac{1}{4} = -0{,}25$$

Set supply and demand equal

$$6 + \frac{3}{3}Q = 20 - \frac{4}{3}Q$$
$$\frac{7}{3}Q = 14$$
$$\boldsymbol{Q^* = 6}$$

Insert Q in either of the functions

$$\boldsymbol{P^* = 6 + 6 = 12}$$

2. Maja consumes goods X and Y. Maja's utility function is $U(X,Y) = 100X^{\frac{1}{2}}Y^{\frac{1}{4}}$.
   a) What is Maja's marginal rate of substitution (MRS_{XY}) if good X is on the horizontal axis and good Y is on the vertical axis?
   b) Maja's income is 150 DKK, the price of good X is 1 DKK, and the price of good Y is 10 DKK. How many units of good X and of good Y does Maja consume in order to maximize utility?
   c) Now suppose the price of good Y changes. With the new prices, we observe that Maja's marginal rate of substitution at the optimal bundle is 1. Does this imply that the price of good Y has gone up or gone down? Explain your answer.

$$MRS = \frac{100 * 0{,}5x^{-0{,}5} * Y^{0{,}25}}{100 * 0{,}25 * X^{0{,}5} * Y^{-0{,}75}} = \frac{0{,}5}{0{,}25} * \frac{Y}{X} = -\frac{2Y}{X}$$

$$150 = 1X + 10Y$$

$$MRT = -\frac{1}{10}$$
$$MRS = MRT$$
$$\frac{2Y}{X} = 0{,}1$$
$$2Y = 0{,}1X$$
$$X = 20Y$$

$$150 = 20Y + 10Y$$
$$150 = 30Y$$
$$Y^* = 5$$
$$X^* = 20 * 5 = 100$$

c) Now suppose the price of good Y changes. With the new prices, we observe that Maja's marginal rate of substitution at the optimal bundle is 1. Does this imply that the price of good Y has gone up or gone down? Explain your answer.

$$\frac{2Y}{X} = \frac{1}{10}$$
$$\frac{Y}{X} = \frac{1P_x}{5P_y}$$

For the optimal bundle to be 1, the price of Y would have to go down, so that it would be $\frac{1}{1} = 1$.

3. Ray's utility function over money is given by $U(M) = M^{1/2}$.
   a) Is Ray risk-averse or risk-loving? Explain your answer using a graph or using equations.
   b) Suppose Ray's initial wealth is $M_0 = 9$ $. Suppose he has the choice of buying a lottery ticket which will cost him 5 $. This lottery will pay 12 $ with probability $\frac{1}{3}$ and pays 0 $ with probability $\frac{2}{3}$. Will Ray buy this lottery? Why or why not?
   c) Now suppose Ray's utility function is $U(M) = M^2$. Will Ray buy the lottery mentioned in part (b) now? Why or why not?

$$U(M) = M^{0,5}$$
$$U(M)' = 0,5M^{-0,5}$$
$$U(M)'' = 0,5 * -0,5 * M^{-1,5} = -0,25M^{-1,5}$$

As the second-order derivative is negative, Ray is risk averse.

$$EU = \frac{1}{3}(9 + 12 - 5)^{0,5} + \frac{2}{3}(4)^{0,5} = 2,67$$
$$U(W_9) = 9^{0,5} = 3$$

He would not buy the ticket

$$U(M) = M^2$$
$$EU = \frac{1}{3}(9 + 12 - 5)^2 + \frac{2}{3}(4)^2 = 96$$
$$U(W_9) = 9^2 = 81$$

$$EU > U(W_9)$$

He will buy the ticket, as he is now risk seeking, and the expected utility is higher than his utility of having his current wealth.

4. Suppose the production function for producing chairs is given by $Q = K^{2/3}L^{2/3}$, where Q is the quantity of chairs.
   a) Suppose the price of capital is $r = 1$ and the price of labour is $w = 4$. If a firm wants to produce 16 chairs, what combination of capital and labor will it use to minimize costs?
   b) Now, suppose in addition to paying for capital and labor the firm will also have to pay fixed lump-sum tax of 20 to the government. Would this affect your answers for part (a)? Why or why not?
   c) Given the production function, how many firms do you think will operate in this market? Explain your answer.

$$Q = K^{0,66} * L^{0,66}$$
$$MRTS = \frac{0,66}{0,66} * \frac{K}{L} = \frac{K}{L}$$
$$|MRTS| = \left|\frac{W}{R}\right|$$
$$\frac{K}{L} = \frac{4}{1}$$
$$K = 4L$$
$$Q = wL + rK$$
$$16 = 4 * L + 1(4L)$$
$$16 = 4L$$
$$\boldsymbol{L^* = 4}$$

$$K = 4L$$
$$\boldsymbol{K^* = 4(4) = 16}$$

c) Tax will not affect the production function as it is only a product of the wage, labour, capital and interest rate.

d) Since it's increased to scale ($\alpha + \beta > 1$), it's not perfect competition at least

5. Consider a market where the demand function is $P = 78 - 2Q$.
   a) If there is monopolist supplying the market, with a total cost function $TC = 10 + 2Q$, then what is the profit maximizing quantity sold and how much profit does the firm earn.
   b) Now suppose another firm enters the market the first firm is a Stackleberg leader. The leader produces quantity Q_1 and the follower produces Q_2. Their total cost functions are $TC_1 = 10 + 2Q_1$ and $TC_2 = 10 + 2Q_2$. How much does the Stackleberg leader produce and how much does the follower produce? What is the new market price?

$$P = 78 - 2Q$$
$$TC = 10 + 2Q$$
$$MR = 78 - 4Q$$
$$MC = 2$$

$$MR = MC$$
$$78 - 4Q = 2$$

$$4Q = 76$$
$$Q^* = 19$$

$$P(Q^* = 19) = 78 - 2(19) = 78 - 38 = 40 = P^*$$

Idk about question b, it's not in the curriculum ☺

Common math rules

Multiplying fractions with a real number

$$\frac{A}{B} * C = \frac{A * C}{B}$$

Multiplying fractions with a fraction

$$\frac{A}{B} * \frac{C}{D} = \frac{A * C}{B * C}$$

$$\frac{3}{5} * \frac{7}{9} = \frac{3 * 7}{5 * 9} = \frac{21}{45}$$

Dividing two fractions

$$\frac{\frac{x}{y}}{\frac{z}{i}} = \frac{x}{y} : \frac{z}{i} = \frac{x}{y} * \frac{i}{z} = \frac{xi}{yz}$$

$$\frac{\frac{1}{3}}{\frac{6}{4}} = \frac{1}{3} : \frac{6}{4} = \frac{1}{3} * \frac{4}{6} = \frac{4}{18}$$

Dividing fractions with a whole number

$$\frac{A}{B} : C = \frac{\frac{A}{C}}{B} = \frac{A:C}{B}$$

$$\frac{4}{8} : 7 = \frac{\frac{4}{8}}{7} = \frac{2}{7}$$

How to take the first order derivative of $\frac{1}{x}$

1) Rewrite the fraction as following

$$f(x) = \frac{1}{x} = x^{-1}$$

2) Take the derivative like you would with an exponent

$$f(x)' = -1 * x^{-2}$$

3) Rewrite it as function

$$f(x)' = -\frac{1}{x^2}$$

The quadratic formular

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

How to do mixed strategies

Robert and Camilla have lost their phones, and must simultaneously decide whether to meet at the Soccer game or at the Opera. The following matrix describes their payoffs.

		Camilla	
		Soccer	Opera
Robert	Soccer	3,1	0,0
	Opera	0,0	1,3

D Calculate the Mixed Strategies Nash Equilibrium.

		Camilla	
		Soccer P	Opera 1 − P
Robert	Soccer	3,1	0,0
	Opera	0,0	1,3

Camilla chooses

$$P = Soccer$$
$$1 - P = Opera$$

Insert the value of the utility she gets from Robert doing either soccer or opera

$$EU_R(S) = 3 * P + (1 - P) * 0 = 3P$$

$$EU_R(O) = 0 * P + (1 - P) * 1 = 1 - P$$

Camilla wants to choose such that $EU_R(S) = EU_R(O)$

$$EU_R(S) = EU_R(O)$$
$$3P = 1 - P$$

Solve for P

$$4P = 1$$
$$P = Soccer = \frac{1}{4} = 25\%$$

Then

$$1 - p = Opera = 1 - \frac{1}{4} = \frac{3}{4} = 75\%$$

Camilla should choose soccer 25% of the time and opera 75% of the time

Robert chooses

$$Q = Soccer$$
$$1 - Q = Opera$$

		Camilla	
		Soccer	Opera
Robert	Soccer (Q)	3,1	0,0
	Opera (1 − Q)	0,0	1,3

$$EU_C(S) = 1 * Q + (1 - Q) * 0 = Q$$
$$EU_C(O) = 0 * Q + (1 - Q) * 3 = 3 - 3Q$$

Robert wants to choose such that $EU_C(S) = EU_C(O)$

$$EU_C(S) = EU_C(O)$$
$$Q = 3 - 3Q$$
$$4Q = 3$$
$$Q = Soccer = \frac{3}{4} = 75\%$$

Then

$$1 - Q = Opera = 1 - \frac{3}{4} = \frac{1}{4} = 25\%$$

Robert should choose soccer 75% of the time and opera 25% of the time for Camilla to be indifferent

The mixed strategy Nash equilibrium is $NE = \left[\left(\frac{1}{4}; \frac{3}{4}\right); \left(\frac{3}{4}; \frac{1}{4}\right)\right]$

How to find pure strategy Nash equilibrium

Question 3

Rebecca and Camilla are going to a party and must choose how to dress without knowing how the other will be dressed. Rebecca has two dresses to choose from (Ultramarine, Dark orange), and Camilla has three dresses to choose from (Lilac, Mango, Red). The simultaneous game can be described by the following matrix.

		Camilla		
		Lilac	Mango	Red
Rebecca	Ultra marine	0, 2	2, 4	2, 0
	Dark orange	4, 0	0, 2	0, 6

When things are crossed out they will be red

A How many Nash Equilibria in pure strategies exist in this game? Find it/them.

Start with one of them (I start with Rebecca but it doesn't matter)

1) If Rebecca chooses ULTRA MARINE, Camilla should choose MANGO → cross out the other two ((UM: Lilac) and (UM: RED)

		Camilla		
		Lilac	Mango	Red
Rebecca	Ultra marine	0, 2	2, 4	2, 0
	Dark orange	4, 0	0, 2	0, 6

2) If Rebecca chooses DARK ORANGE, Camilla should choose RED → cross out the other two ((DA: Lilac) and (UM: MANGO)

		Camilla		
		Lilac	Mango	Red
Rebecca	Ultra marine	0, 2	2, 4	2, 0
	Dark orange	4, 0	0, 2	0, 6

Now we're done with Rebecca and move on to Camilla

3) If Camilla chooses LILAC, Rebecca should choose DARK ORANGE (cross out UM)

		Camilla		
		Lilac	Mango	Red
Rebecca	Ultra marine	0, 2	2, 4	2, 0
	Dark orange	4, 0	0, 2	0, 6

4) If Camilla chooses Mango, Rebecca should choose ULTRA MARINE (cross out DA)

	Camilla			
Rebecca		Lilac	Mango	Red
	Ultra marine	0, 2	2, 4	2, 0
	Dark orange	4, 0	0, 2	0, 6

5) If Camilla chooses RED, Rebecca should choose ULTRA MARINE (cross out DA)

	Camilla			
Rebecca		Lilac	Mango	Red
	Ultra marine	0, 2	2, 4	2, 0
	Dark orange	4, 0	0, 2	0, 6

We're done and the pure strategy Nash equilibrium is $NE = (Ultra\ marine; mango)$

(((If everything is crossed out; move on to mixed strategy)))

If you were to draw it by hand an easy way to do so is this