Applied Microeconomics

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International business and politics
Exam questions solving guide

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Applying supply and demand model

Consumer has (inverse) demand function, derive price elasticity of demand

1. If asked to derive the price elasticity of demand without any values (P and Q), follow the following formula

$$\varepsilon = -b \cdot \frac{P}{Q}$$

2. Now if given the inverse demand function, rearrange it to the normal demand function (Q=)
3. The numeric value before a multiplying sign ($Q = 20 - P$) is the "b" value, therefore the answer to deriving the price elasticity of demand is

$$\varepsilon = -1 \cdot \frac{P}{Q}$$

If asked to calculate price elasticity of demand when numeric values for P and Q are given (Q=100, P=1)

1. Enter given Q and P values into formula after step 3

$$\varepsilon = -1 \cdot \frac{1}{100}$$

2. Result is the answer to the question

If asked to find the price, when given a price elasticity of demand

1. Insert your given price elasticity of demand (-3) in the expression you derived earlier

$$-3 = -1 \cdot \frac{P}{Q}$$

$$\frac{P}{Q} = 3$$

2. Insert the demand function (Q=) into q's spot and solve for P

3. The result is the price at a given price elasticity of demand

Applying consumer choice

Utility function - calculate optimal consumption and utility for consumer (optimal bundle)

1. Find the MRS with following formula

$$MRS = -\frac{\frac{\delta U}{\delta X}}{\frac{\delta U}{\delta Y}}$$

2. Find the MRT with the following formula

$$MRT = -\frac{P_X}{P_Y}$$

3. Set MRS equal to MRT and solve for x (an expression with both x and y will appear)
4. Enter the expression for x into the following budget constraint

$$income = P_X \cdot X + P_Y \cdot Y$$

5. Solve for Y, which now gives a pure expression (one variable)
6. Enter pure expression for y into expression for x from step 3.
7. Now a value for both X and Y will appear, this is the optimal consumption

If asked to calculate utility for the consumer:

1. Enter values for X and Y into the given utility function
2. The result is the answer

If the two goods are perfect substitutes

1. If the MRS is constant (a number, without any letters), then the goods are perfect substitutes, which means that the elasticity of substitution is infinite
2. Calculating the optimal bundle now depends on the ration between MRS and the MRT
3. Notice that we are looking at the pure numbers, meaning that if there are negative values, the "minus" must be ignored

$$MRS > \frac{P_X}{P_Y}, consume\ only\ good\ X$$

$$MRS < \frac{P_X}{P_Y}, consume\ only\ good\ Y$$

$$MRS = \frac{P_X}{P_Y}, all\ corresponding\ bundles\ are\ optimal$$

If asked what the optimal demand as a function of an unknown is

1. A utility function given could be

$$U(B,S) = B^{\frac{2}{3}} \cdot S^{\frac{1}{3}}$$

2. Because the function is a Cobb Douglas, we can see that the person spends 2/3 of his budget on burgers and 1/3 on pizza. What he spends can be written as the price of burgers times the quantity of burgers

$$P_B \cdot Q_B = \frac{2}{3}M$$

3. The same is done with pizza

$$P_S \cdot Q_S = \frac{1}{3}M$$

4. Now insert values for prices and solve for Q, this is the optimal demand as a function of M

Firms, production, and costs

What is a firm's VC, AC, MC, AVC under perfect competition?

1. The VC is the variable costs and are found by looking at which parts of the TC function includes a variable, E.g.

$$TC = 5Q^2 + 10Q + 5$$

$$VC = 5Q^2 + 10Q$$

2. The AC is the average costs and are found by dividing all parts of the TC with Q

$$AC = \frac{5Q^2 + 10Q + 5}{Q} = 5Q + 10 + \frac{5}{Q}$$

3. The MC is the marginal cost and are found by taking the first order of derivatives of the TC

$$AC' = 5Q^2 + 10Q + 5 = 10Q + 10$$

4. The AVC is the average variable costs, which are calculated by dividing the variable costs with Q

$$AVC = \frac{VC}{Q} = \frac{5Q^2 + 10Q}{Q} = 5Q + 10$$

Firm operating under perfect competition, a TC function and P is given, calculate optimal production

1. The MC function is found by taking the derivatives of the TC function
2. Then because it is a perfect competitive market, the price is the MR, and is set equal to the MC

$$P = MC$$

3. Values are put in, and solve for Q
4. The number of Q's is the optimal level of production

If asked what the optimal level of production is in the long run

1. Find the AC

$$AC = \frac{TC}{Q}$$

2. Differentiate the AC
3. Set the AC' equal to 0
4. Isolate new Q, this is the long run optimal production

If asked how many firms operate in the market, long run, perfect competition

1. Make sure you have the aggregated inverse demand function (p=)
2. Insert your long run optimal production quantity into the AC function to find the long run price
3. Insert the new price into the inverse demand function and solve for Q
4. Now you know how many things are produced totally in the market, divide this number with the amount the single firm from earlier produced, this gives your number of firms operating in the market.

If asked to derive the firm's supply curve

1. A firm in perfect competition has a supply curve equal to its marginal cost curve
2. Take the first order derivatives of the TC functions, this is the supply curve

If asked where the firm minimizes its marginal cost

1. Take the derivatives of the MC functions and set equal to 0
2. Solve for Q, this is the level of production that minimizes marginal cost

If asked what the aggregate supply curve in the long run

1. Find the optimal price in the long run - set the optimal Q into the AC function
2. The price found is the aggregated supply curve, because there is free market entry under perfect competition

A production function for capital and labor is provided - calculate $MP_L\ and\ MP_K$

1. To find marginal product of labor, take the partial derivatives to first labor e.g.

$$Q = K^{0,25} \cdot L^{0,75}$$

$$MP_L = K^{0,25} \cdot 0,75^{-0,25}$$

2. Now to find the marginal product of capital, take the partial derivatives to capital

$$Q = K^{0,25} \cdot L^{0,75}$$

$$MP_K = 0,25K^{-0,75} \cdot L^{0,75}$$

If you are asked what the optimal demand for capital and labor, based on a price and a quantity

1. The optimal demand is where marginal product of labor divided with marginal product of capital equals the two prices divided by each other

$$\frac{MP_L}{MP_K} = \frac{P_L}{P_K}$$

2. Now insert given values and isolate for L, your expression will include all the variables

$$\frac{0{,}75}{0{,}25} \cdot \frac{K}{L} = \frac{3K}{L}$$

$$\frac{3K}{L} = \frac{P_L}{P_K}$$

$$P_L \cdot L = P_K \cdot 3K$$

3. Now insert the values for income (the q and p given) and the new expression for $P_L \cdot L$ in the budget constraint

$$1000 = 3P_K \cdot K + P_K \cdot K$$

$$\frac{1000}{4P_K} = K$$

$$K = \frac{250}{P_K}$$

4. Now insert the expression for K in the general budget constraint

$$1000 = P_L \cdot L + P_K \cdot K$$

$$1000 = P_L \cdot L + P_K \cdot \frac{250}{P_K}$$

$$1000 = P_L \cdot L + 250$$

$$\frac{750}{P_L} = L$$

5. The optimal demand for capital and labor and the expressions found $K = and\ P =$

If asked what the optimal demand for capital and labor is, and receiving values for $P_K = 5\ and\ P_L = 10$

1. Insert the values for the prices into the expression found in the previous question

$$K = \frac{250}{5} = 50$$

$$L = \frac{750}{10} = 75$$

Several production functions are provided, and you are asked to explain and prove whether there is constant return to scale, decreasing return to scale or increasing return to scale

1. Explain what constant return to scale means

$$F(aK, aL) = aF(K, L)$$

2. Test whether the given production functions lives up to this

$$Q = L + K$$

$$F(aK, aL) = aL + aK = a \cdot (L + K), constant\ returns\ to\ scale$$

$$Q = L \cdot K$$

$$F(aK, aL) = aL \cdot aK = a^2 \cdot (L \cdot K) > a \cdot F(K, L), when\ a > 1, increasing\ returns\ to\ scale$$

$$Q = L^{0,5} \cdot K^{0,5}$$

$$F(aK, aL) = (aL)^{0,5} \cdot (aK)^{0,5} = a \cdot (L^{0,5} \cdot K^{0,5}) = a \cdot F(F, L), constant\ returns\ to\ scale$$

Applying the competitive market

Finding market equilibrium when given aggregated demand and supply

1. Make sure that both the supply and demand are not invert, hence they are $Q =$

2. Set them equal and solve for P e.g. $Q_D = 120 - 10P \text{ and } Q_S = 2P$

$$20 - 10P = 2P$$

$$12P = 120$$

$$P = 10$$

3. Now you know the price, insert it into the P spot in the demand function

$$Q_D = 120 - 10 \cdot 10 = 20$$

4. Your values for Q and P are the market equilibrium

If asked what the Consumer surplus and producer surplus are

1. Consumer surplus can be found by first finding the inverse demand function

$$P_D = 12 - \frac{1}{10}Q$$

2. The inverse demand function tells us what the maximum price the consumer will pay, 12

3. The consumer surplus is calculated as the area of the triangle from the maximum price 12 to the actual price 10 and the quantity supplied 20

$$CS = \frac{1}{2} \cdot (12 - 10) \cdot 20 = 20$$

4. In practice, PS is the area of the triangle that lies under the equilibrium price, bounded by the supply curve itself. The length of this triangle is the equilibrium quantity q∗. The height of that triangle is the difference between the equilibrium price p∗, and the intercept of the inverse supply function. So, if inverse supply function is $P = \frac{1}{2} \cdot Q$ then

$$PS = \frac{1}{2} \cdot Q^* \cdot (intercept\ inverse\ supply)$$

$$PS = \frac{1}{2} \cdot (10 - 0) \cdot 20 = 100$$

If asked about producer surplus in the long run in a perfect competitive market

1. It is always 0, because there are no fixed costs in the long run and the profit of a producer is always 0 in the long run in a perfect competitive market

If asked what the producer surplus is the short run, with no fixed costs

1. The producer surplus is the same as profit when there are no fixed costs

$$PS = TR - TC$$

If a tax is introduced, either on consumer or producer

1. If the tax is on the producer, rewrite the supply function to include the tax (If asked to find the equilibrium price and quantity in a perfectly competitive market, it does not matter who the tax is levied on)

$$P_S = \frac{1}{2}Q + 3$$

2. Find the new equilibrium by setting the new inverse supply equal to the inverse demand and solve for Q

3. Insert the found value for Q into the newly found after tax inverse supply function to find the new price

4. The new values for Q and P are the new after-tax equilibrium

If asked what the welfare cost of the tax is (deadweight loss)

1. Use the same walkthrough guide as when calculating consumer surplus and producer surplus

2. Substitute the new sales prices and quantities in and remember to subtract the tax from the producer's welfare

$$CS = \frac{1}{2} \cdot (12 - 10{,}5) \cdot 15 = 11{,}25$$

$$PS = \frac{1}{2} \cdot (10{,}5 - 3) \cdot 15 = 56{,}25$$

3. The deadweight loss is calculated with this formula

$$DWL = \Delta CS + \Delta PS + \Delta TR$$

4. Using the example from earlier

$$DWL = (11{,}25 - 20) + (56{,}25 - 100) + 45 = -7{,}5$$

Monopolies

Calculating optimal level of production and price for monopolies

1. Change a demand function into an inverse demand function (P=)
2. To find the MR for the monopoly, double the numeric value before the letter e.g.

$$P = 10 - \frac{1}{2}Q \rightarrow P = 10 - Q$$

3. Now set the MR equal to the MC, which can be given, if not take the derivatives of the TC function, which is the MC

$$MR = MC$$

4. Solve for Q, which is the optimal level of production
5. To find the optimal price, insert the optimal Q value into the first function of price

If asked what the profit for the monopoly is

1. Insert the new optimal number of productions into the inverse demand function (P=), to find an optimal price
2. Multiply the optimal price with the optimal number of productions to get TR

$$TR = P \cdot Q$$

3. Insert the optimal quantity into the total cost function
4. Subtract the total cost from the total revenue

$$Profit = TR - TC$$

If asked what the introduction of a tax does to the optimal level of output and profit for a monopolist, where all tax is on the consumer

1. Alter the demand function to now include + tax on the P side e.g.

$$P = 10 - 0{,}5Q$$

$$P + 2 = 10 - 0{,}5Q$$

2. Find the new MR by isolating P and doubling the numeric value before the letter

$$P = 8 - Q$$

3. Set the new MR equal to the old MC and isolate for Q, this is the new optimal production
4. Insert the new optimal production into the new inverse demand function to find the new price
5. Calculate the profit with following formula

$$profit = TR - TC = Q \cdot P - (TC(Q)$$

Price discrimination

If asked what the optimal price, quantity and profit are in two different countries, where price discrimination is possible

1. If the marginal cost is constant, same for both countries, the firm operates in the two countries as they were separate and calculate quantity, price and profit just as a normal monopoly.

2. The profit is the two maximized profits from both countries added together

If asked what the optimal price, quantity and profit are when the company cannot price discriminate

1. Treat the two markets as one, adding each market demands together, E.g.

$$Q_{DK} = 10 - 0{,}2P$$

$$Q_{UK} = 20 - 0{,}2P$$

$$Q = 30 - 0{,}4P$$

2. No convert it to the inverse demand function and follow the regular way to find a monopoly's optimal quantity, price and profit

Risk

Deriving first- and second order derivatives for risk aversion, for what values of risk is risk neutral, risk averse or risk seeking

1. First derive the first order e.g.

$$U(Y) = Y^r$$

$$U(Y') = r \cdot Y^{r-1}$$

2. Derive the second order

$$U(Y'') = r \cdot (r-1) \cdot Y^{r-2}$$

3. Now conclude that this utility is negative when $r < 1$, positive when $r > 1$ and 0 when $r = 1$
4. This means that the person is risk averse when $r < 1$, risk neutral when $r = 1$ and risske seeking when $r > 1$

Willingness to pay (insurance) and premium (utility function given)

1. First find the expected utility gained by using the formula

$$EU = \% \cdot U(value) + \% \cdot U(value)$$

2. If the EU when buying insurance is greater than not buying insurance, he will buy insurance
3. Now the utility function (e.g. $U(Y) = Y^{0,5}$) is set equal to the EU

$$Y^{0,5} = EU$$

4. Solve for Y, and that is the willingness to pay / certainty equivalent
5. To find the risk premium, first calculate EV with this formula

$$EV = \% \cdot value + \% \cdot value$$

6. Now subtract the willingness to pay from the expected value

$$Premium = EV - Y$$

If asked what the mean and standard deviation of a lottery

1. When asked what the mean is, calculate the variance

$$Var = \% \cdot (value - EV)^2 + \% \cdot (value - EV)^2$$

2. The standard deviation can be calculated by

$$Standard\ deviation = \sqrt{Var}$$

Assessing whether functions are risk averse, neutral, or increasing

1. Looking at the three utility functions

$$U(W) = \exp(W)$$

$$U(W) = \ln(W)$$

$$U(W) = W$$

2. (i) The exponential function is convex, and the person is risk seeking
   (ii) The logarithmic function is concave and the person is risk averse
   (iii) The linear function implies that the person is risk neutral